TWO KINDS OF VACILLATION IN ROTATING LABORATORY EXPERIMENTS

RICHARD L. PFEFFER

Florida State University, Tallahassee, Fla.

and

Y. CHIANG

Lamont Geological Observatory,[1] Columbia University, Palisades, N.Y.

ABSTRACT

Two types of vacillation are distinguished in a rotating differentially heated annulus of fluid: (1) Tilted-trough vacillation, which has been discussed in a number of previous papers in the literature, is characterized by periodic changes in the wave shapes, in which the waves first tilt "north-west" to "south-east", and later "north-east" to south-west", relative to the rotating annulus. (2) Amplitude vacillation, which was first observed by the authors in a series of student demonstrations conducted at the Lamont Geological Observatory, is characterized by periodic expansions and contractions of the wave pattern with no noticeable change in the tilt of the disturbances. It is suggested that the details of the energy conversions are different in the two cases. Some speculations are also presented concerning the role of viscosity in limiting the number of degrees of freedom available for vacillation.

1. BACKGROUND

For certain ranges of the thermal Rossby number (Ro_T) and an appropriately defined Taylor number (Ta), quasi-geostrophic wave motions can be produced in a rotating annulus of fluid by imposing an axially symmetrical gradient of heating in the radial direction. In the usual experimental arrangement, the fluid is heated differentially by conduction through outer and inner metal cylinders which are in contact with hot and cold constant-temperature baths. Details of such laboratory experiments and their meteorological significance have been discussed by Hide [4, 5, 6], Fultz et al. [3], and Fowlis and Hide [2]. Following Fowlis and Hide we shall adopt the definitions

$$Ro_T \equiv \frac{gH\beta(T_s-T_n)}{\Omega^2(R_s-R_n)^2}, \tag{1}$$

$$Ta \equiv \frac{4\Omega^2(R_s-R_n)^5}{\nu^2 H}, \tag{2}$$

where T_s and T_n are the outer and inner bath temperatures, respectively, g is the acceleration of gravity, H is the depth of the fluid, β is the coefficient of volume expansion, Ω is the angular velocity of the rotating annulus, R_s and R_n are the radii of the outer and inner cylindrical walls, and ν is the kinematic viscosity. It is significant that the rotational static stability number

$$s \equiv \frac{(\beta\overline{T})(gH)}{\Omega^2 R_m(R_s-R_n)}\left(\frac{H}{\overline{T}}\frac{\overline{\Delta T}}{\Delta z}\right), \tag{3}$$

which occurs in theoretical treatments (see, for example, Eady [1]; Kuo [7]; Pfeffer [10]) varies in proportion to the imposed thermal Rossby number in these experiments. Here, $\overline{T}$ is the mean temperature of the fluid, $\overline{\Delta T/\Delta z}$ is the mean lapse rate and

$$R_m \equiv \tfrac{1}{2}(R_s+R_n).$$

Fultz, Hide, and their collaborators have identified four different regimes of convection in a rotating, differentially heated annulus of fluid, as shown schematically in figure 1. The steady wave regime is distinguished by the fact that

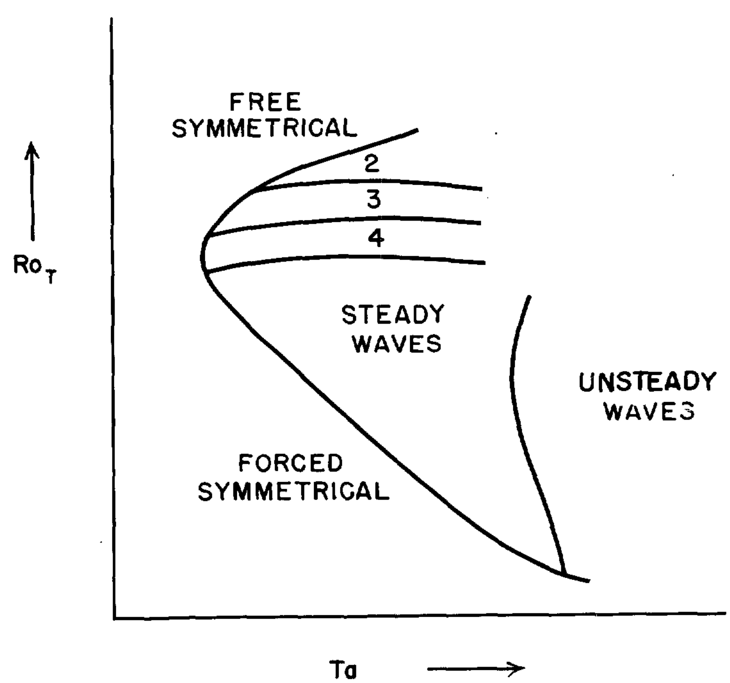

FIGURE 1.—Schematic picture showing the four regimes identified by Fultz et al. [3] and by Fowlis and Hide [2].

[1] Lamont Geological Observatory Contribution Number 1016.

only one wave number at a time dominates the flow, the higher wave numbers occurring at lower values of Ro_T. These waves are superficially quasi-steady and regular in appearance. The unsteady wave regime is characterized by more turbulent, continually changing flow patterns (similar to the large-scale motions of the earth's atmosphere) in which a spectrum of waves of different scale dominates the flow. The axially symmetrical regimes are Hadley circulations, the lower one being forced and the upper one free, as discussed by Kuo [7, 8]. In the region between the steady wave regime and the unsteady wave regime Hide, Fultz, and their collaborators have observed a phenomenon which Hide called "vacillation," in which the waves undergo a periodic change first tilting from "north-west" to "south-east" and later from "north-east" to "south-west" relative to the annulus. An example of such a vacillation cycle is shown in figure 2. Practically all of the discussion in the literature to date concerning vacillation centers around this phenomenon (although Fultz has also noted the existence of wave-number vacillation at the boundaries between two different "steady" wave patterns). Fultz et al. [3] showed quantitatively that the radial eddy flux of angular momentum, which is proportional to the covariance $\overline{u'v'}$, changes sign during a tilted trough vacillation cycle. Here, u and v are the zonal (azimuthal) and meridional (radial) components of velocity, respectively. The bar over a quantity represents a zonal average and the prime represents the departure at a point from this average. Fultz's data can also be used to show that the rate of conversion of kinetic energy between the eddies and the mean zonal flow ($K_E \rightarrow K_Z$), which depends upon the covariance of $\overline{u'v'}$ and $\partial(\bar{u}/R)/\partial R$, undergoes substantial fluctuations in magnitude, and apparently also in sign, during this cycle. Here, K_E is the eddy kinetic energy, K_Z is the kinetic energy of the mean zonal flow, and R is radial distance from the axis of rotation of the annulus. Because of the characteristics of this vacillation it seems appropriate to call it "tilted-trough vacillation" or "kinetic energy vacillation," although it is recognized that fluctuations in the rate of transformation between two forms of kinetic energy in a quasi-geostrophic, baroclinic fluid must be accompanied by fluctuations in the rate of conversion between potential and kinetic energy.

2. AMPLITUDE VACILLATION

In the course of conducting student demonstrations of annulus convection at the Lamont Geological Observatory, Columbia University, during the winter of 1962–63 the present writers observed, at certain values of Ro_T and Ta, still a different kind of "vacillation" in which the wave pattern expands and contracts with no change in the wave number and no noticeable change in the tilt of the disturbances. Two examples of this type of vacillation in a 4-wave pattern are shown in figures 3 and 4. In both cases the phenomenon may be described as a "square-to-lobed wave" vacillation, or an "amplitude" vacillation, although it is recognized that a constant-amplitude disturbance superimposed upon a zonal current which fluctuates in intensity could give the same appearance. Since the writers could find no previous pictures or discussion of this particular kind of vacillation in the published literature,[2] it was considered desirable to prepare the present note calling attention to this phenomenon and discussing the characteristic differences between it and the tilted-trough vacillation discussed by Hide, Fultz, and their collaborators.

Before proceeding with the discussion, it is appropriate to say a word about the apparatus and the accuracy of the data. The apparatus consisted of equipment which was readily available from around the Observatory or could be purchased locally at low cost. Thus, for example, a 4-speed record player was acquired for use as a rotating turntable. A small heating coil connected to a variac voltage control was suspended in the outer bath in a fixed position such that it heated and stirred the bath as the annulus rotated by it. In this way the outer bath was maintained at a uniform high temperature. Tap water circulating through the central bath through rubber tubing served to maintain this bath at a uniform cold temperature. Mercury-in-glass thermometers immersed in the inner and outer baths were monitored almost continuously throughout each experiment.

It was found that fluctuations of the hot and cold bath temperatures could be kept within a range of 0.5° C. over the period of the experiment. In experiments with sufficiently large imposed bath temperature differences these fluctuations represent a small percentage error. According to equation (1), the temperature difference required to attain a given thermal Rossby number increases as we increase the size of the annular gap within which the working fluid is contained (other things being held constant). For the purpose of the present series of experiments the annular gap was chosen to be 12.4 cm. With this size annulus and a rotation rate of 3.1 sec.$^{-1}$, the temperature difference needed to produce the conditions shown in figures 3 and 4 is of the order of 40° to 50° C. The fluctuations of the bath temperatures, therefore, contribute only a 1 percent error to the temperature difference ($T_s - T_n$). Although the variation of the fluid properties (β, ν, etc.) with temperature over such a large range is not insignificant, it is difficult to believe that the phenomenon we observed was due to these variations. Amplitude vacillation and its characteristic changes with Ro_T have, in fact, been confirmed more recently by a careful series of experiments conducted at Florida State University by the writers' colleague, Dr. William Fowlis. One such experiment, in which this phenomenon was produced in a

[2] Prof. D. Fultz has since informed us that he, too, has observed this phenomenon and reported it in his Final Report under Contract AF 19(604)–8361 (April 1964), figure 38 B. The phenomenon was also observed by Dr. W. Fowlis in connection with his Ph. D. thesis research (M.I.T., January 1964). More recently, the existence of amplitude vacillation has been confirmed by Professor R. Hide ("On the Dynamics of Rotating Fluids and Related Topics in Geophysical Fluid Dynamics," *Bulletin of the American Meteorological Society*, vol. 47, No. 11, Nov. 1966).

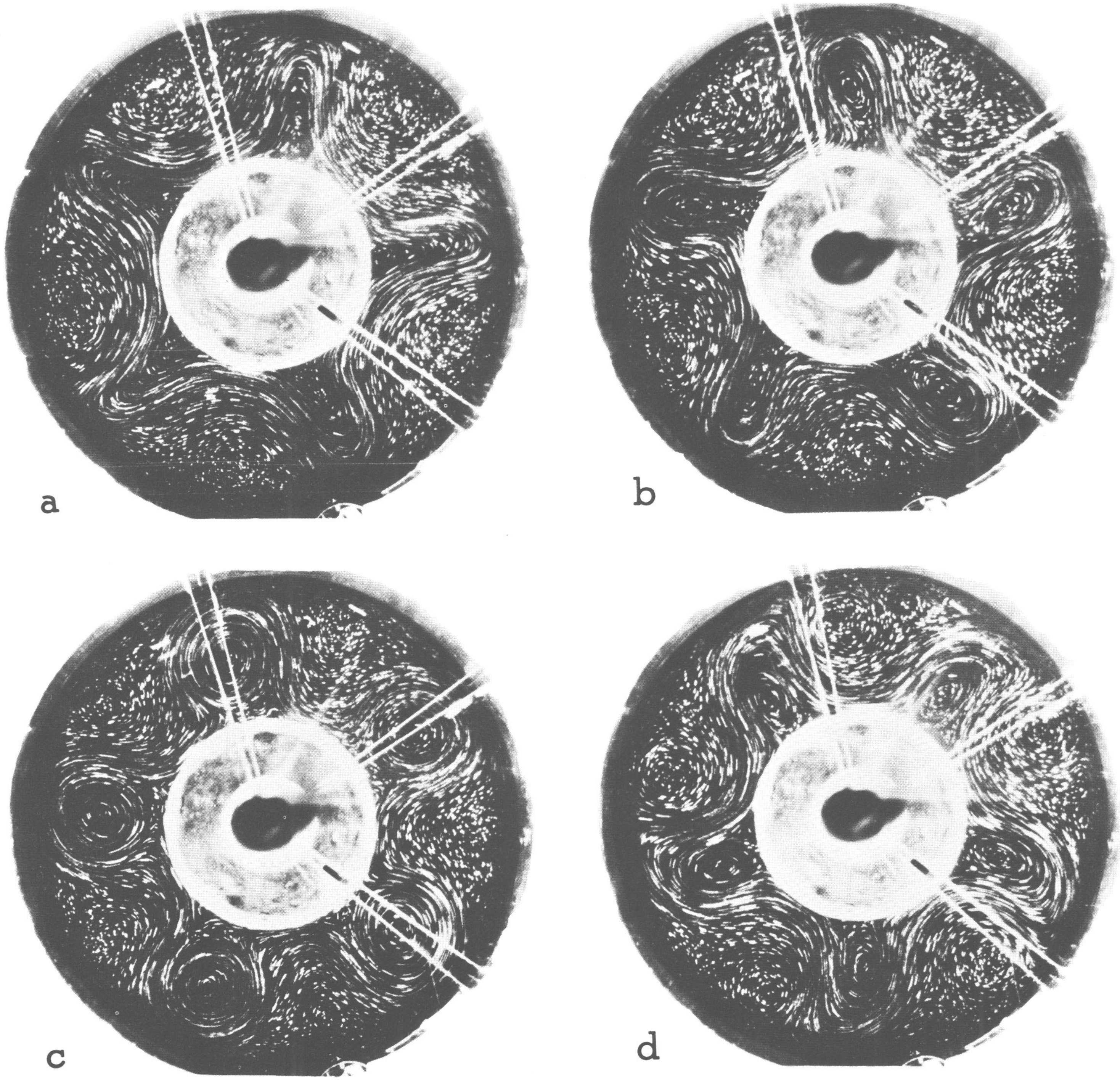

FIGURE 2.—A sequence of four photographs, furnished by Professor Dave Fultz of the University of Chicago, showing the top surface flow in a rotating annulus at different stages of tilted-trough vacillation. (The rotation here is counterclockwise.)

smaller annulus ($R_s-R_n=2.55$ cm.) with a smaller, thermostatically controlled bath temperature difference ($T_s-T_n=12.5°$ C.) was reported by Pfeffer, Mardon, Sterbenz, and Fowlis [11].

Having chosen a large annular gap we can adjust the Taylor number (equation (2)) to the range in which other investigators have worked by using a viscous fluid. The fluid used for the Lamont experiments was Medicol mineral oil purchased from a local drug store. A Saybolt viscometer was used to determine the viscosity of the oil at different temperatures and an analytical balance was used to determine the coefficient of volume expansion. The experimental parameters and nondimensional numbers for the experiments shown in figures 3 and 4 are listed as follows: For both experiments, $R_s=20.0$ cm., $R_n=7.6$ cm., $\Omega=3.1$ sec.$^{-1}$, $H=6$ cm., $\beta=6\times10^{-4}$ deg.$^{-1}$,

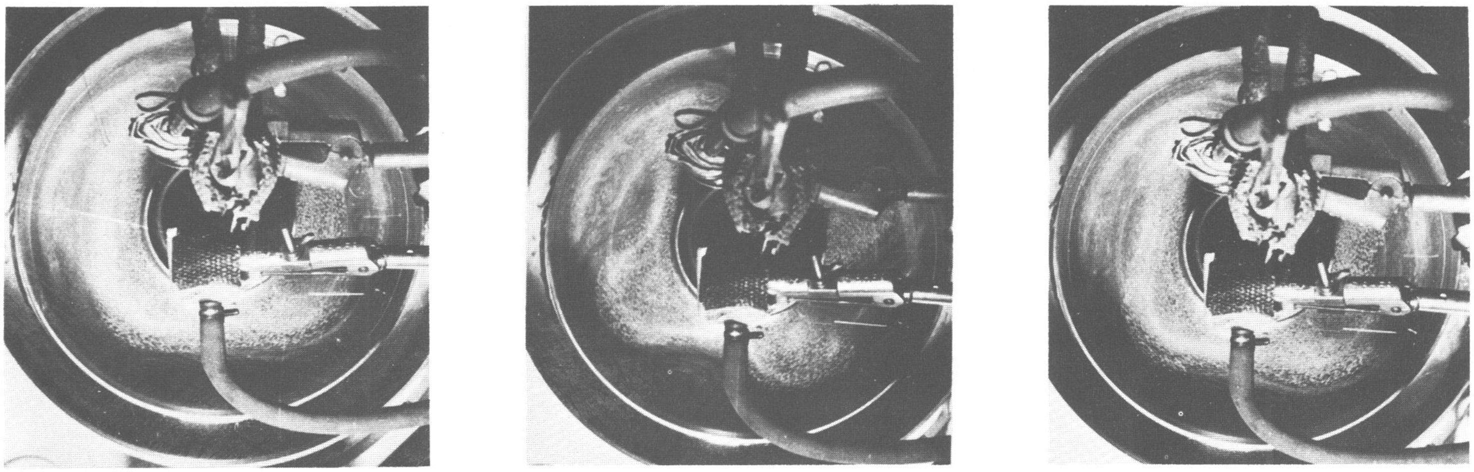

FIGURE 3.—A sequence of three photographs, obtained by the present authors at the Lamont Geological Observatory April 24, 1963, showing the top surface flow at the extremes of "amplitude" vacillation. The fluid used for the experiment was mineral oil with aluminum powder mixed in as a tracer. Experimental conditions: $R_s=20.0$ cm., $R_n=7.6$ cm., $\Omega=3.1$ sec.$^{-1}$, $H=6$ cm., $\beta=6\times10^{-4}$ deg.$^{-1}$, $\nu=2\times10^{-2}$ cm.^{2} sec.$^{-1}$, $T_s=52.5°$ C., $T_n=13.7°$ C., $Ta=6\times10^7$, $Ro_T=0.095$. (The rotation here is clockwise.)

$\nu=2\times10^{12}$ cm.^{12} sec.$^{-1}$, $Ta=6\times10^7$. For the April 24, 1963 experiment $T_s=52.5°$ C., $T_n=13.7°$ C., $Ro_T=0.095$. For the May 17, 1963 experiment $T_s=65°$ C., $T_n=16°$ C.; $Ro_T=0.12$.

It is significant that the intensity of the vacillation is greater in the May 17, 1963 experiment, apparently because of the greater value of Ro_T. This result has been confirmed by Dr. Fowlis' more recent experiments which show that if a steady wave pattern has been attained, amplitude vacillation will take place with increasing intensity as Ro_T is increased. The striking similarity between this phenomenon and certain index breakdowns in the earth's atmosphere is shown by the comparison in figure 5. The main difference between index fluctuations in the atmosphere and amplitude vacillation in laboratory experiments is that the former are sporadic whereas the latter are more closely cyclic in nature. Winston and Krueger [12] have shown that the index breakdown pictured in figure 5 is associated with a rapid adiabatic conversion of potential energy into kinetic energy of the atmosphere. The similarity of the two phenomena shown in figure 5, and the absence of a noticeable change in the tilt of the disturbances in the annulus during amplitude vacillation, suggest that amplitude vacillation also involves large adiabatic conversions from potential to kinetic energy which vary periodically in intensity. To contrast this phenomenon with tilted-trough, kinetic-energy vacillation, it seems appropriate to call it "potential energy vacillation." Since both types of vacillation no doubt involve some variation in each of the links in the energy cycle, it is probable that the distinction is more a matter of degree than of kind.

The transformation from steady waves to intense amplitude vacillation as the thermal Rossby number is increased probably results from differences in the efficiency of each wave scale in converting potential to kinetic energy at different values of the imposed temperature gradient and rotation rate. As discussed by Eady [1] and Kuo [9], the efficiency of this energy conversion depends upon the ratio of the angle in the x–z plane through which a parcel of fluid moves ($\delta z/\delta x\equiv W/V$) to the slope of a potential temperature surface ($\partial z/\partial x\equiv(\partial T/\partial x)/(\partial T/\partial x)$). This ratio depends upon the rotational static stability number (s) which, in these experiments, is a function of the imposed temperature gradient and rotation rate.

3. VACILLATION AND THE ENERGY CYCLE

The above considerations suggest that it would be useful to study vacillation in light of the energy cycle of the fluid. For the purpose of discussing the characteristic differences between potential energy vacillation and kinetic energy vacillation and comparing these with steady state conditions, the energy cycle pictured in figure 6 contains sufficient detail. A more elaborate cycle would be needed for discussions of wave number vacillation, minor wave vacillation, and other phenomena which involve transfers of energy among different wave numbers. In the figure, A_Z and A_E are the zonal and eddy components of the available potential energy, K_Z and K_E are the zonal and eddy components of the kinetic energy, G_Z and G_E are the zonal and eddy components of the rate of generation of available potential energy, and F_Z and F_E are the zonal and eddy components of the rate of dissipation of kinetic energy; the quantities $(A_Z\rightarrow A_E)$, $(A_E\rightarrow K_E)$, $(K_E\rightarrow K_Z)$, and $(K_Z\rightarrow A_Z)$ are rates of conversion from one form of energy to another. If the wave pattern were truly steady, the rates of conversion of energy from one form to another

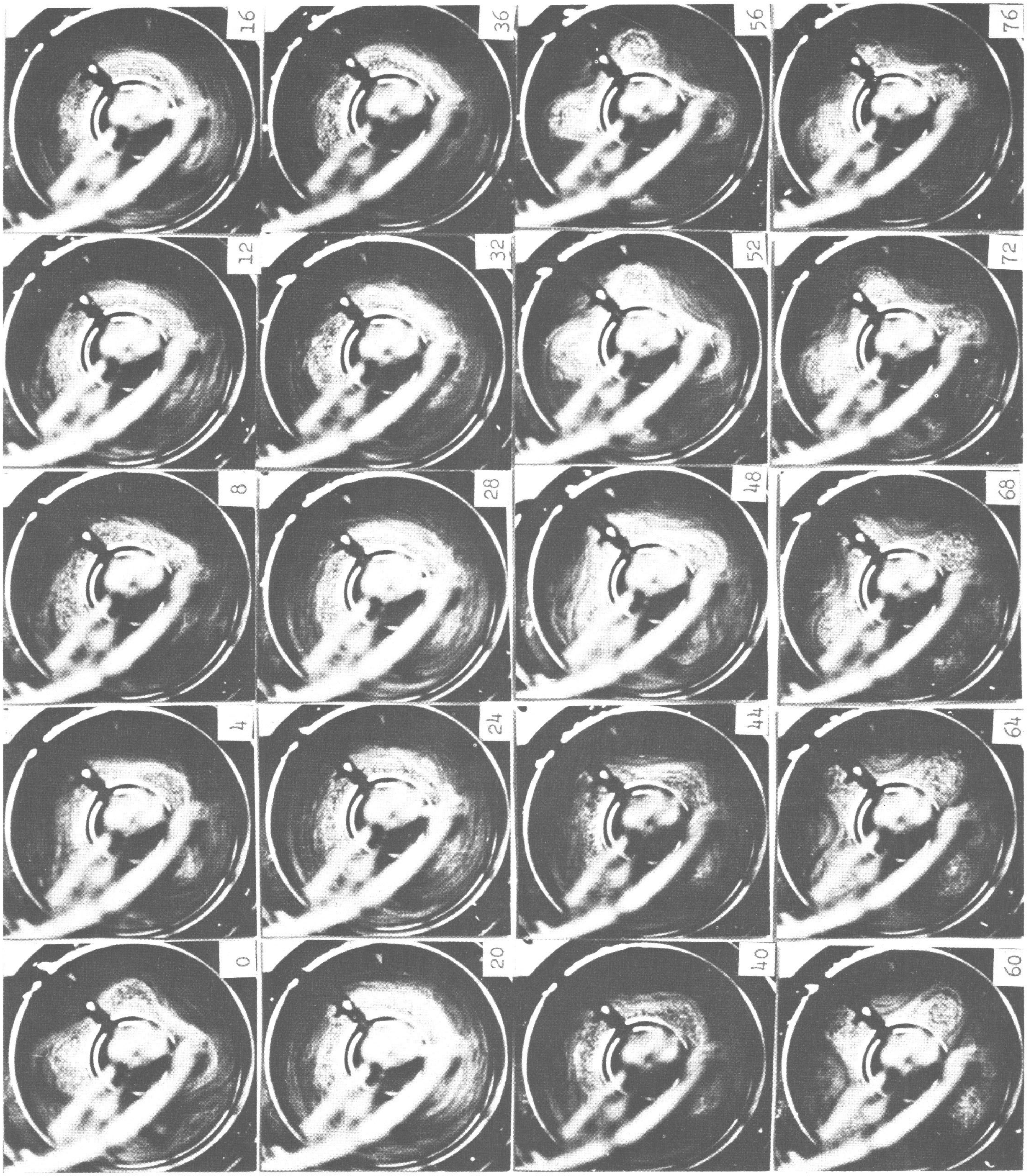

FIGURE 4.—Photographs obtained by the present authors at the Lamont Geological Observatory May 17, 1963, showing a sequence of stages of "amplitude" vacillation. With illustration rotated 90° clockwise for viewing, the numbers in the lower right-hand corner of each photo indicate the number of rotations of the turntable after the first picture was taken. Note the explosive growth of the waves from rotation 44 to rotation 60. This appears to be a more intense vacillation than the one shown in figure 3 which was conducted at a smaller value of Ro_T. Experimental conditions: $R_s=20.0$ cm., $R_n=7.6$ cm., $\Omega=3.1$ sec.$^{-1}$, $H=6$ cm., $\beta=6\times10^{-4}$ deg.$^{-1}$, $\nu=2\times10^{-1}$ cm.^{2} sec.$^{-1}$, $T_s=65°$ C., $T_n=16°$ C., $Ta=6\times10^7$, $Ro_T=0.12$. (The rotation here is clockwise.)

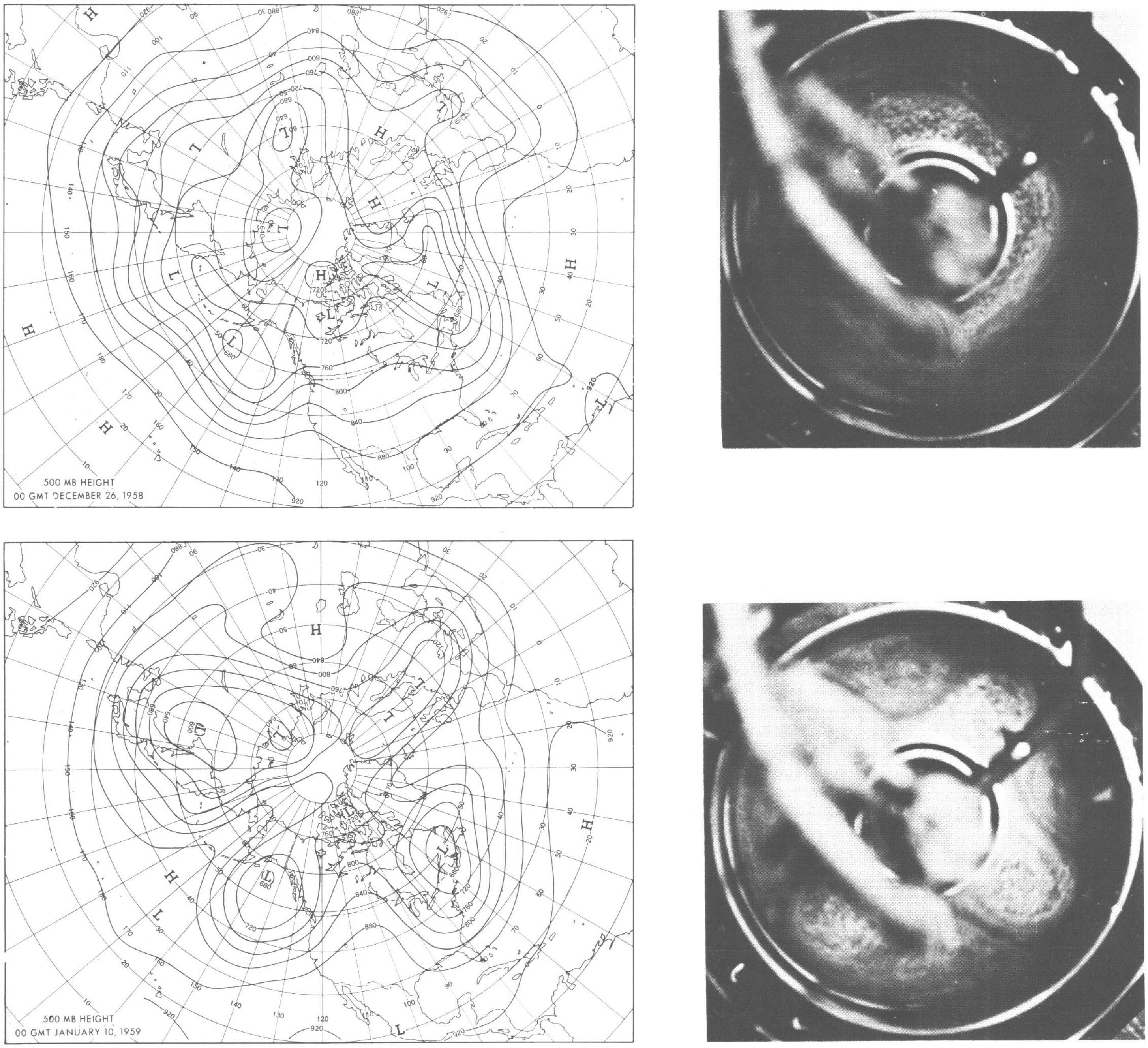

FIGURE 5.—500-mb. charts showing two stages of an atmospheric index cycle (after Winston and Krueger [12]) and top surface photos showing two stages of "amplitude" vacillation (rotations 36 and 60 in fig. 4).

would be in balance such that there would be no variation in the amounts of energy stored in each of the component forms. Intuitively, one feels that a perfect balance among the rates of energy transformation must be a rare occurrence. Since each of the links in the energy cycle depends upon a different function of the temperature, pressure, and velocity, we should expect, more generally, at least temporary unbalances among the different rates of energy transformation. This would lead to variations in the amount of energy stored in each of the component forms and, therefore, to vacillation or irregular flow. Careful examination of the flow in rotating laboratory experiments reveals that vacillation of one form or another, and in one degree of intensity or another, is more common than steady-state conditions.

Some characteristics of amplitude vacillation are suggested by the sequence of pictures shown in figure 4. The apparent disappearance of large-amplitude wave cyclones during the first 24 rotations, for example, suggests that the adiabatic rates of energy conversion ($A_Z \rightarrow A_E$) and ($A_E \rightarrow K_E$), decline sharply during this period. If G_Z is sufficiently large, it follows that A_Z (and, therefore, also the radial temperature gradient) increases, making conditions favorable by the end of this period for a more rapid release of available potential engergy and a corresponding increase in the eddy kinetic energy. The growth of the wave cyclones, which takes place slowly during rotations 28 to 40, becomes more explosive during rotations 44 to 60. During the latter period, the adiabatic rates of energy conversion, $(A_Z \rightarrow A_E)$ and $(A_E \rightarrow K_E)$, probably are the dominant processes affecting A_Z, A_E, and K_E. As the eddies grow, however, the supply of available potential energy diminishes and these conversions must again decrease in intensity. Remembering that the events shown in figure 4 are cyclic, so that rotations 0 to 24 follow rotations 60 to 76, it is suggested that sometime after rotation 60, and continuing through rotation 24, $(A_Z \rightarrow A_E)$ becomes smaller than G_Z and $(A_E \rightarrow K_E)$ becomes smaller than F_E, leading to an increase in A_Z and a decrease in K_E during this period.

Preliminary experiments by W. W. Fowlis at Florida State University suggest that amplitude vacillation takes place well within the "steady wave" regime and that it is most pronounced at lower Taylor numbers, higher thermal Rossby numbers, and higher Prandtl numbers. The experiments by R. Hide and D. Fultz suggest that tilted-trough vacillation takes place at somewhat larger Taylor numbers, particularly in the vicinity of the boundary between steady and unsteady waves. The latter phenomenon appears to involve significant variations in the intensity and sign of the energy transformation $(K_E \rightarrow K_Z)$ as well as other energy conversion terms, suggesting that barotropic stability or instability is a factor to be considered in discussing the energetics of tilted-trough vacillation. At even higher Taylor numbers the waves are more turbulent and unsteady resembling more closely the wave cyclones and anticyclones found in the earth's atmosphere.

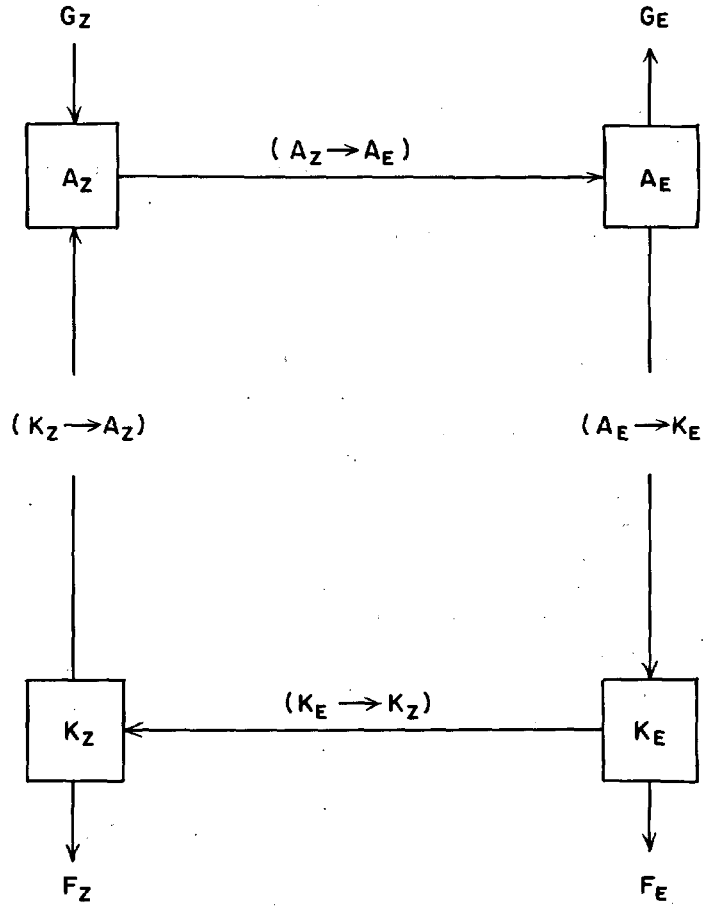

FIGURE 6.—Schematic picture of the energy cycle in a differentially heated, rotating annulus of fluid.

4. SOME SPECULATIONS

The above observations suggest the possible importance of viscosity in modifying the behavior of the waves which are formed by baroclinic instability. In the absence of dissipation, waves of all scales are baroclinically unstable. For each value of the rotation rate and mean static stability, there is a particular wave scale which grows faster than both longer and shorter waves. The scale of the most unstable wave increases with increasing static stability and decreasing rotation rate. In the absence of dissipation, therefore, a spectrum of waves centered on the most unstable wavelength can be expected to grow. The waves which develop in this way are free to interact with one another as well as with the mean "zonal" current. Experimentally, at high Taylor numbers, where the effect of dissipation is small, we do indeed find a spectrum of waves continually interacting with one another to form an unsteady wave pattern. In a more viscous fluid, on the other hand, only the most unstable waves grow fast enough to appear. The greater the viscosity of the fluid (or, the lower the Taylor number), the narrower should be the observed spectrum of waves. At sufficiently low values of the Taylor number, no waves are unstable and the only possible motion is a forced symmetrical circulation. At slightly larger values, all but a single two-dimensional wave number are damped by viscosity. This corresponds to the left-hand side of the region of "steady waves" in figure 1. Here, vacillation can result only from baroclinic interactions between the waves and the mean zonal current involving energy conversions of the kind $(A_Z \rightarrow A_E)$ and $(A_E \rightarrow K_E)$. The size of this region on the diagram, and the intensity of the vacillations, must depend upon the Prandtl number. We have suggested that "amplitude vacillation" is brought about by such energy conversions and that differences in the efficiency of a given wave scale at different values of the rotational static stability number determine where on the diagram the waves will be steady and where they will vacillate.

We may speculate further that at slightly larger values of the Taylor number a given wave scale in the azimuthal direction can have more than one wave number in the radial direction. Under these conditions, the wave is free to interact barotropically (as well as baroclinically) with the mean zonal current, leading to energy conversions of the form $(K_E \rightarrow K_Z)$, in addition to other possible energy transformations. Near the boundary between steady waves and unsteady waves, where dissipation is less important, we do in fact observe the phenomenon we have called tilted-trough vacillation which involves such interactions. At even larger Taylor numbers, more wave scales become unstable, allowing more interactions, and unsteady flow is observed. Tracing this sequence of observations in the reverse order, one gets the impression that the effect of increasing the viscosity of the fluid is to narrow the spectrum of the unstable waves, allowing only the fastest growing waves to appear, and thereby to reduce the number of degrees of freedom available for vacillation.

These conclusions should, of course, be considered tentative, pending our ability to obtain synoptic data from rotating laboratory experiments in sufficient detail to measure the appropriate energy transformation integrals. Although synoptic data distributions can be obtained in steady wave regimes by moving a small number of probes from point to point as the experiment proceeds and translating time information into space information, this technique is no longer feasible for studying vacillating and irregular wave regimes. Instead, new techniques will have to be devised in order to obtain truly synoptic distributions of temperature and fluid velocity in such experiments. The importance of studying the time variations of the energy cycle in vacillating regimes justifies a major effort to obtain such measurements. It is suggested that a synoptic network of observations in a large rotating annulus, which would accomplish in the laboratory what a global data network will accomplish in the atmosphere, is both necessary and feasible at this stage of the development of the atmospheric and planetary sciences.

ACKNOWLEDGMENTS

The experiments reported in this paper were performed at the Lamont Geological Observatory under Office of Naval Research Contract Nonr 266(70). Further experimentation and interpretation of the results were accomplished at Florida State University under Office of Naval Research Contract Nonr 988(14) and U.S. Weather Bureau Grant WBG–45.

The writers are grateful to Prof. Dave Fultz of the University of Chicago for providing us with his unpublished photos of tilted-trough vacillation shown in figure 2.

REFERENCES

1. E. T. Eady, "Long Waves and Cyclone Waves," *Tellus*, vol. 1, No. 3, Aug. 1949, pp. 33–52.
2. W. W. Fowlis and R. Hide, "Thermal Convection in a Rotating Annulus of Liquid: Effect of Viscosity on the Transition Between Axisymmetric and Nonaxisymmetric Flow Regimes," *Journal of the Atmospheric Sciences*, vol. 22, No. 5, May 1965, pp. 541–558.
3. D. Fultz, R. Long, G. Owens, W. Bohan, R. Kaylor, and J. Weil, "Studies of Thermal Convection in a Rotating Cylinder With Some Implication for Large-Scale Atmospheric Motions," *Meteorological Monographs*, vol. 4, No. 21, 1959, pp. 1–104.
4. R. Hide, "Some Experiments on Thermal Convection in a Rotating Liquid," *Quarterly Journal of the Royal Meteorological Society*, vol. 19, No. 339, Jan. 1953, p. 161.
5. R. Hide, "Fluid Motions in the Earth's Core and Some Experiments on Thermal Convection in a Rotating Liquid," *Proceedings of the First Symposium on the Use of Models in Geophysical Fluid Dynamics, September 1953*, U.S. Office of Naval Research, Washington, D.C., 1956, pp. 101–116.
6. R. Hide, "An Experimental Study of Thermal Convection in a Rotating Fluid," *Philosophical Transactions, Royal Society of London*, Series A, vol. 250, 1958, pp. 441–478.
7. H. L. Kuo, "Forced and Free Axially-Symmetric Convection Produced by Differential Heating in a Rotating Fluid," *Journal of Meteorology*, vol. 13, No. 6, Dec. 1956, pp. 521–527.
8. H. L. Kuo, "Forced and Free Meridional Circulations in the Atmosphere," *Journal of Meteorology*, vol. 13, No. 6, Dec. 1956, pp. 561–568.
9. H. L. Kuo, "Energy Releasing Processes and Stability of Thermally Driven Motions in a Rotating Fluid," *Journal of Meteorology*, vol. 13, No. 1, Feb. 1956, pp. 82–101.
10. R. L. Pfeffer, "Available Potential Energy in a Quasi-Geostrophic Atmosphere," *Journal of the Atmospheric Sciences*, 1967 (in press).
11. R. L. Pfeffer, D. Mardon, P. Sterbenz, and W. Fowlis, "A New Concept of Available Potential Energy," *Proceedings of the International Symposium on the Dynamics of Large-Scale Processes*, Moscow, June 1965.
12. J. Winston and A. Krueger, "Some Aspects of a Cycle of Available Potential Energy," *Monthly Weather Review*, vol. 89, No. 9, Sept. 1961, pp. 307–318.

[*Received September 19, 1966; revised December 1, 1966*]